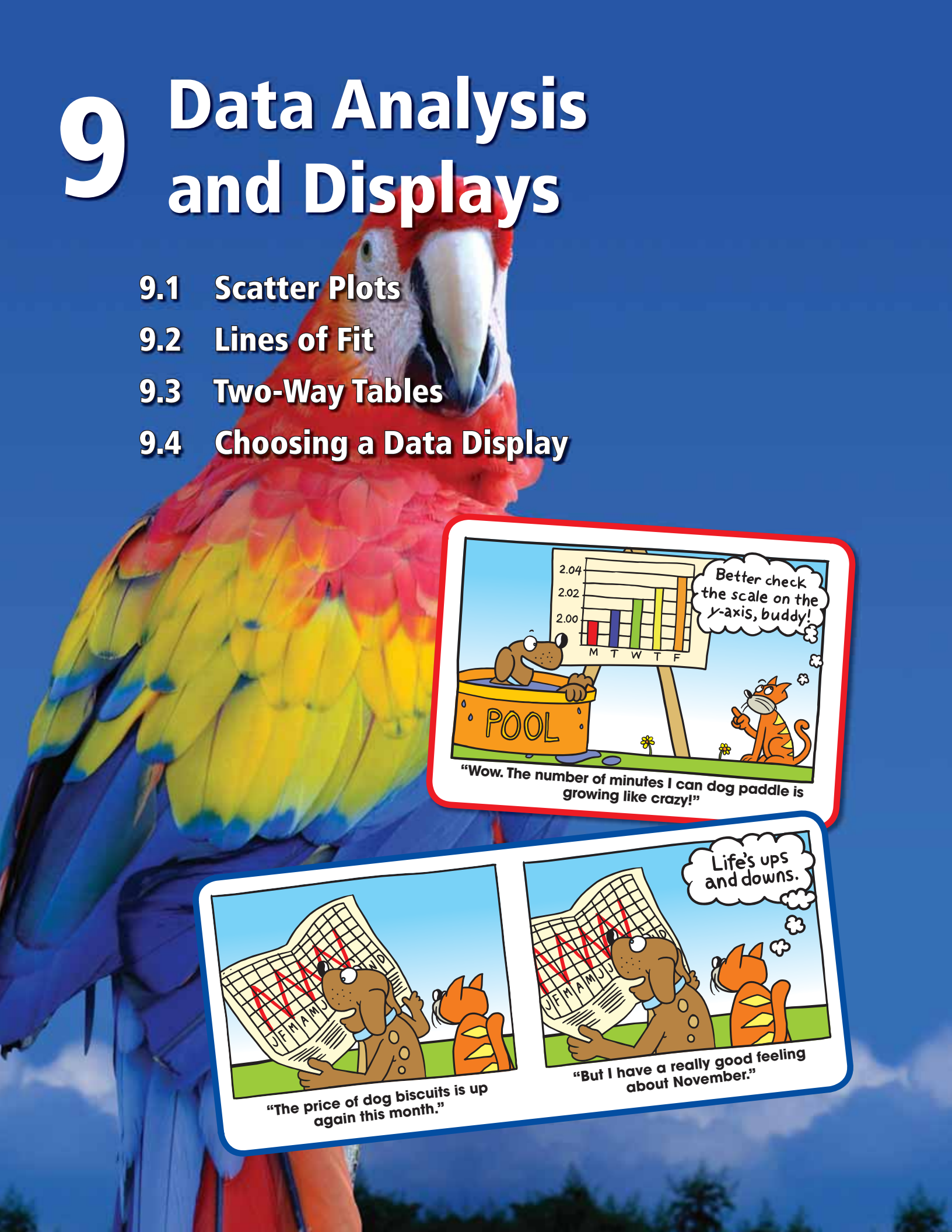
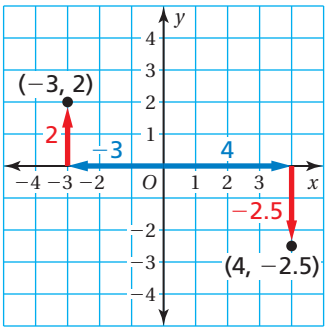
9 Data Analysis and Displays



What You Learned Before

Plotting Points (6.NS.6c)

Example 1 **Plot (a) (−3, 2) and (b) (4, −2.5) in a coordinate plane. Describe the location of each point.**



a. Start at the origin. Move 3 units left and 2 units up. Then plot the point.

∴ The point is in Quadrant II.

b. Start at the origin. Move 4 units right and −2.5 units down. Then plot the point.

∴ The point is in Quadrant IV.

Try It Yourself

Plot the ordered pair in a coordinate plane. Describe the location of the point.

1. (1, 3) **2.** (−2, 4) **3.** (1, −3.5) **4.** $\left(-1\frac{3}{4}, -2\frac{1}{4}\right)$

Writing an Equation Using Two Points (8.F.4)

Example 2 **Write in slope-intercept form an equation of the line that passes through the points (4, 2) and (−1, −8).**

Find the slope:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-8 - 2}{-1 - 4} = \frac{-10}{-5} = 2$$

Then use the slope $m = 2$ and the point (4, 2) to write an equation of the line.

$y - y_1 = m(x - x_1)$	Write the point-slope form.
$y - 2 = 2(x - 4)$	Substitute 2 for m, 4 for x_1, and 2 for y_1.
$y - 2 = 2x - 8$	Distributive Property
$y = 2x - 6$	Write in slope-intercept form.

Try It Yourself

Write in slope-intercept form an equation of the line that passes through the given points.

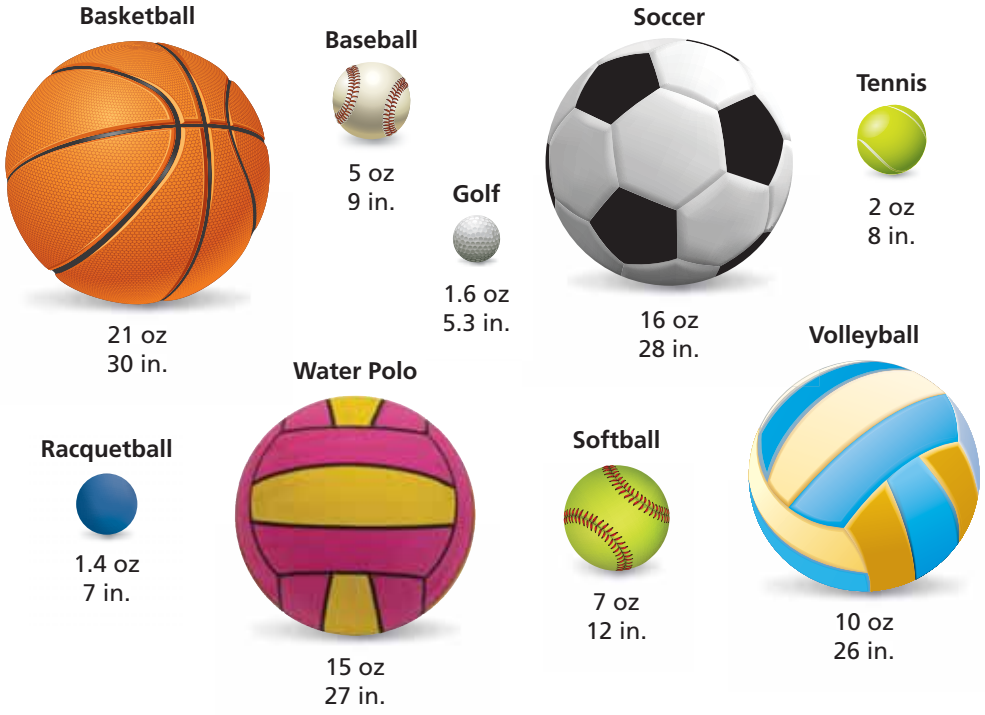
5. (−1, 4), (3, 8) **6.** (0, −1), (−8, −2) **7.** (6, 8), (3, −9)

9.1 Scatter Plots

Essential Question How can you construct and interpret a scatter plot?

1 ACTIVITY: Constructing a Scatter Plot

Work with a partner. The weights x (in ounces) and circumferences C (in inches) of several sports balls are shown.

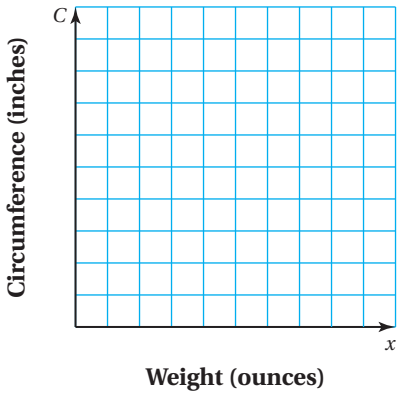


a. Choose a scale for the horizontal axis and the vertical axis of the coordinate plane shown.

b. Write the weight x and circumference C of each ball as an ordered pair. Then plot the ordered pairs in the coordinate plane.

c. Describe the relationship between weight and circumference. Are any of the points close together?

d. In general, do you think you can describe this relationship as *positive* or *negative*? *linear* or *nonlinear*? Explain.



e. A bowling ball has a weight of 225 ounces and a circumference of 27 inches. Describe the location of the ordered pair that represents this data point in the coordinate plane. How does this point compare to the others? Explain your reasoning.

Common Core

Data Analysis

In this lesson, you will

- construct and interpret scatter plots.
- describe patterns in scatter plots.

Learning Standard 8.SP.1

2 ACTIVITY: Constructing a Scatter Plot

Math Practice 5

Recognize Usefulness of Tools

How do you know when a scatter plot is a useful tool for making a prediction?

Work with a partner. The table shows the number of absences and the final grade for each student in a sample.

Absences	Final Grade
0	95
3	88
2	90
5	83
7	79
9	70
4	85
1	94
10	65
8	75

a. Write the ordered pairs from the table. Then plot them in a coordinate plane.

b. Describe the relationship between absences and final grade. How is this relationship similar to the relationship between weight and circumference in Activity 1? How is it different?

c. **MODELING** A student has been absent 6 days. Use the data to predict the student's final grade. Explain how you found your answer.

3 ACTIVITY: Identifying Scatter Plots

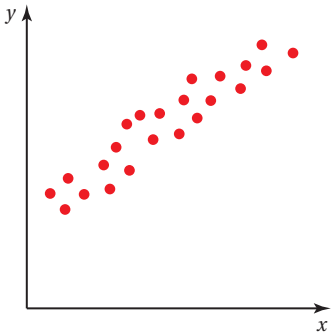
Work with a partner. Match the data sets with the most appropriate scatter plot. Explain your reasoning.

a. month of birth and birth weight for infants at a day care

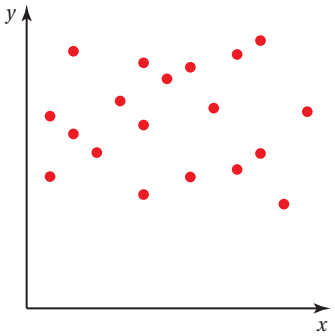
b. quiz score and test score of each student in a class

c. age and value of laptop computers

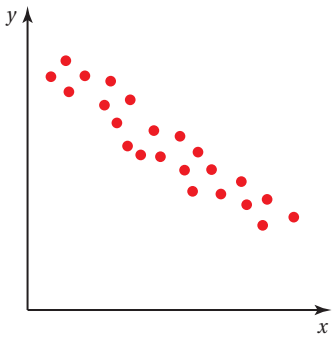
i.



ii.



iii.



What Is Your Answer?

4. How would you define the term *scatter plot*?

5. **IN YOUR OWN WORDS** How can you construct and interpret a scatter plot?

Practice Use what you learned about scatter plots to complete Exercise 7 on page 376.

9.1 Lesson

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Lesson Tutorials
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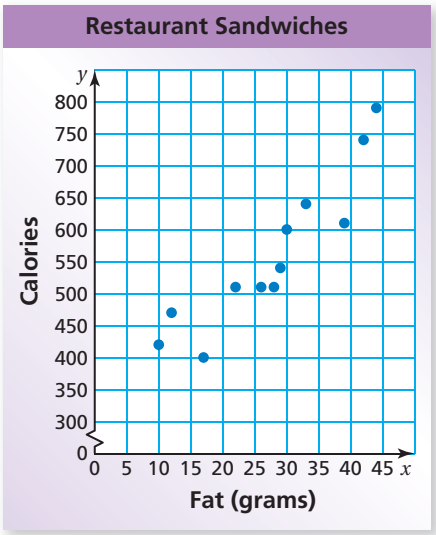
Key Vocabulary
scatter plot, *p. 374*

Key Idea

Scatter Plot

A **scatter plot** is a graph that shows the relationship between two data sets. The two sets of data are graphed as ordered pairs in a coordinate plane.

EXAMPLE 1 Interpreting a Scatter Plot

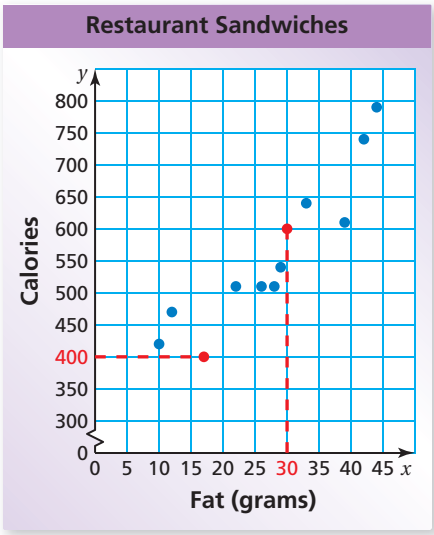


The scatter plot at the left shows the amounts of fat (in grams) and the numbers of calories in 12 restaurant sandwiches.

a. How many calories are in the sandwich that contains 17 grams of fat?

Draw a horizontal line from the point that has an x-value of 17. It crosses the y-axis at 400.

∴ So, the sandwich has 400 calories.



b. How many grams of fat are in the sandwich that contains 600 calories?

Draw a vertical line from the point that has a y-value of 600. It crosses the x-axis at 30.

∴ So, the sandwich has 30 grams of fat.

c. What tends to happen to the number of calories as the number of grams of fat increases?

Looking at the graph, the plotted points go up from left to right.

∴ So, as the number of grams of fat increases, the number of calories increases.

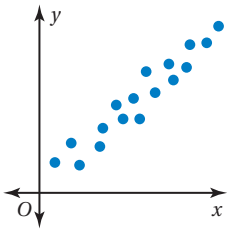
On Your Own

Now You're Ready
Exercises 8 and 9

1. **WHAT IF?** A sandwich has 650 calories. Based on the scatter plot in Example 1, how many grams of fat would you expect the sandwich to have? Explain your reasoning.

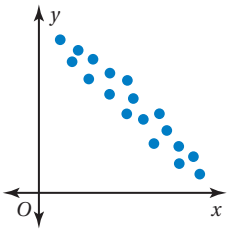
A scatter plot can show that a relationship exists between two data sets.

Positive Linear Relationship



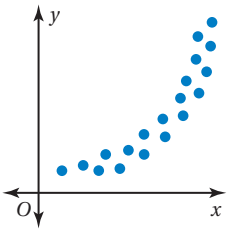
The points lie close to a line. As x increases, y increases.

Negative Linear Relationship



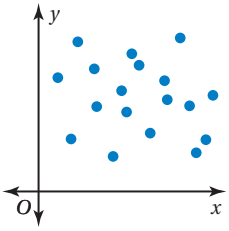
The points lie close to a line. As x increases, y decreases.

Nonlinear Relationship



The points lie in the shape of a curve.

No Relationship

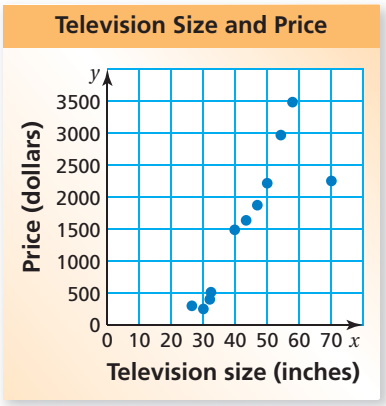


The points show no pattern.

EXAMPLE 2 Identifying Relationships

Describe the relationship between the data. Identify any outliers, gaps, or clusters.

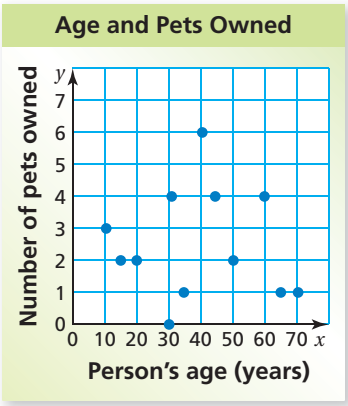
a. television size and price



The points appear to lie close to a line. As x increases, y increases.

So, the scatter plot shows a positive linear relationship. There is an outlier at (70, 2250), a cluster of data under \$500, and a gap in the data from \$500 to \$1500.

b. age and number of pets owned



The points show no pattern.

So, the scatter plot shows no relationship. There are no obvious outliers, gaps, or clusters in the data.

On Your Own

Now You're Ready
Exercises 10–12

2. Make a scatter plot of the data and describe the relationship between the data. Identify any outliers, gaps, or clusters.

Study Time (min), x	30	20	60	90	45	10	30	75	120	80
Test Score, y	80	74	92	97	85	62	83	90	70	91

9.1 Exercises

Check It Out
Help with Homework
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Vocabulary and Concept Check

1. **VOCABULARY** What type of data do you need to make a scatter plot? Explain.
2. **REASONING** How can you identify an outlier in a scatter plot?

LOGIC Describe the relationship you would expect between the data. Explain.

3. shoe size of a student and the student's IQ
4. time since a train's departure and the distance to its destination
5. height of a bouncing ball and the time since it was dropped
6. number of toppings on a pizza and the price of the pizza

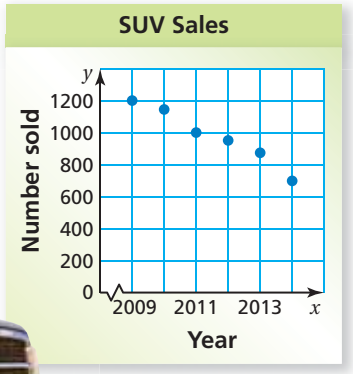
Practice and Problem Solving

7. **JEANS** The table shows the average price (in dollars) of jeans sold at different stores and the number of pairs of jeans sold at each store in one month.

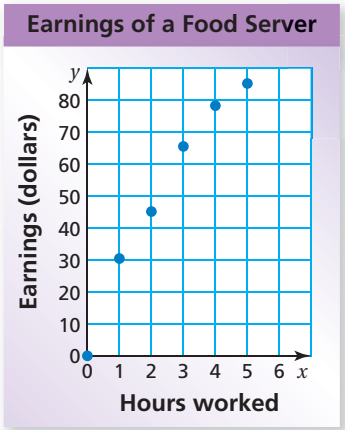
Average Price	22	40	28	35	46
Number Sold	152	94	134	110	81

   a. Write the ordered pairs from the table and plot them in a coordinate plane.
   b. Describe the relationship between the two data sets.

8. **SUVS** The scatter plot shows the numbers of sport utility vehicles sold in a city from 2009 to 2014.



   a. In what year were 1000 SUVs sold?
   b. About how many SUVs were sold in 2013?
   c. Describe the relationship shown by the data.

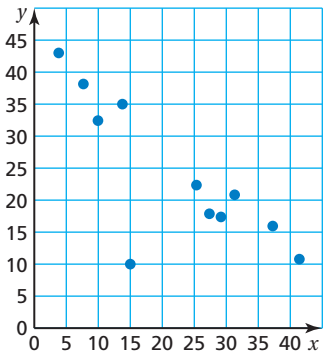


9. **EARNINGS** The scatter plot shows the total earnings (wages and tips) of a food server during one day.

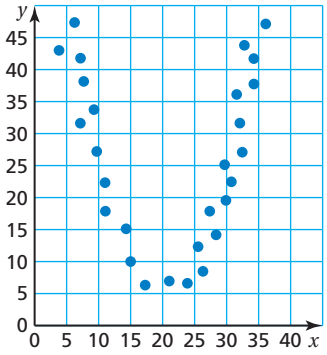
   a. About how many hours must the server work to earn $70?
   b. About how much did the server earn for 5 hours of work?
   c. Describe the relationship shown by the data.

Describe the relationship between the data. Identify any outliers, gaps, or clusters.

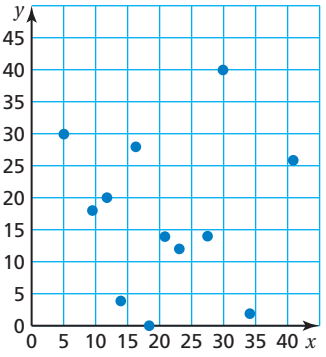
10.



11.



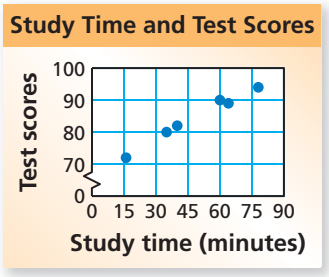
12.



13. HONEY The table shows the average price per pound for honey in the United States from 2009 to 2012. What type of relationship do the data show?

Year, x	2009	2010	2011	2012
Average Price per Pound, y	$4.65	$4.85	$5.15	$5.53

14. TEST SCORES The scatter plot shows the numbers of minutes spent studying and the test scores for a science class. (a) What type of relationship do the data show? (b) Interpret the relationship.



15. OPEN-ENDED Describe a set of real-life data that has a negative linear relationship.

16. PROBLEM SOLVING The table shows the memory capacities (in gigabytes) and prices (in dollars) of 7-inch tablet computers at a store. (a) Make a scatter plot of the data. Then describe the relationship between the data. (b) Identify any outliers, gaps, or clusters. Explain why you think they exist.

Memory (GB), x	8	16	4	32	4	16	4	8	16	8	16	8
Price (dollars), y	200	230	120	250	100	200	90	160	150	180	220	150

17. Reasoning Sales of sunglasses and beach towels at a store show a positive linear relationship in the summer. Does this mean that the sales of one item *cause* the sales of the other item to increase? Explain.

Fair Game Review What you learned in previous grades & lessons

Use a graph to solve the equation. Check your solution. *(Section 5.4)*

18. $5x = 2x + 6$

19. $7x + 3 = 9x - 13$

20. $\frac{2}{3}x = -\frac{1}{3}x - 4$

21. MULTIPLE CHOICE When graphing a proportional relationship represented by $y = mx$, which point is not on the graph? *(Section 4.3)*

Ⓐ $(0, 0)$ Ⓑ $(0, m)$ Ⓒ $(1, m)$ Ⓓ $(2, 2m)$

9.2 Lines of Fit

Essential Question How can you use data to predict an event?

1 ACTIVITY: Representing Data by a Linear Equation

Work with a partner. You have been working on a science project for 8 months. Each month, you measured the length of a baby alligator.

The table shows your measurements.

	September							April
Month, x	0	1	2	3	4	5	6	7
Length (in.), y	22.0	22.5	23.5	25.0	26.0	27.5	28.5	29.5

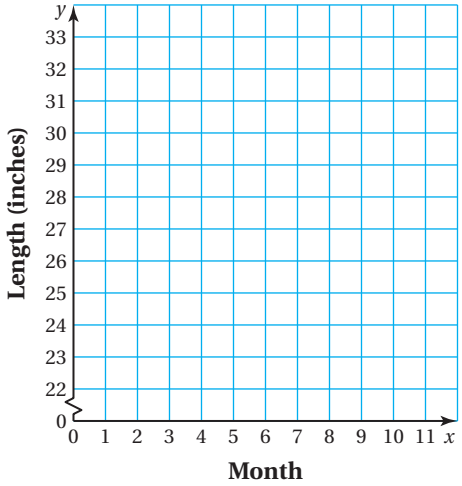
Use the following steps to predict the baby alligator's length next September.

a. Graph the data in the table.

b. Draw a line that you think best approximates the points.

c. Write an equation for your line.

d. **MODELING** Use the equation to predict the baby alligator's length next September.



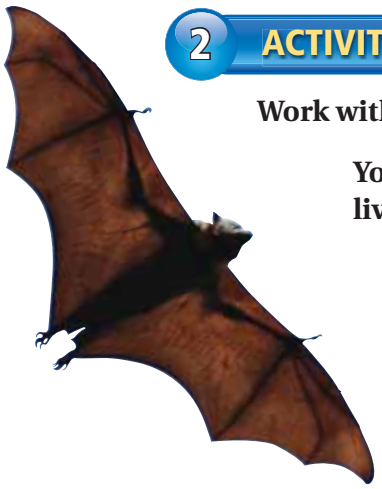
Common Core

Data Analysis

In this lesson, you will

- find lines of fit.
- use lines of fit to solve problems.

Learning Standards
8.SP.1
8.SP.2
8.SP.3



2 ACTIVITY: Representing Data by a Linear Equation

Work with a partner. You are a biologist and study bat populations.

You are asked to predict the number of bats that will be living in an abandoned mine after 3 years.

To start, you find the number of bats that have been living in the mine during the past 8 years.

The table shows the results of your research.

Math Practice 4

Use a Graph

How can you draw a line that "fits" the collection of points? How should the points be positioned around the line?

7 years ago → this year

Year, x	0	1	2	3	4	5	6	7
Bats (thousands), y	327	306	299	270	254	232	215	197

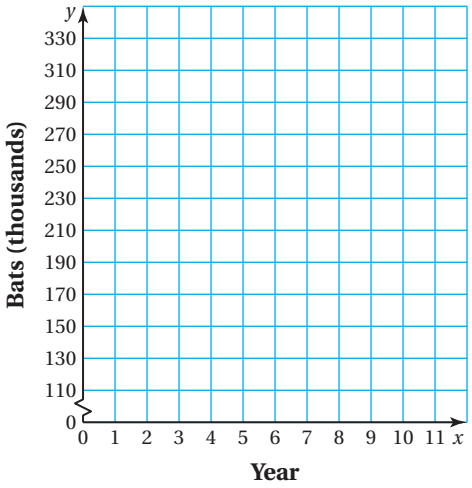
Use the following steps to predict the number of bats that will be living in the mine after 3 years.

a. Graph the data in the table.

b. Draw a line that you think best approximates the points.

c. Write an equation for your line.

d. **MODELING** Use the equation to predict the number of bats in 3 years.



What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you use data to predict an event?

4. **MODELING** Use the Internet or some other reference to find data that appear to have a linear pattern. List the data in a table, and then graph the data. Use an equation that is based on the data to predict a future event.

Practice Use what you learned about lines of fit to complete Exercise 4 on page 382.

9.2 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
line of fit, *p. 380*
line of best fit, *p. 381*

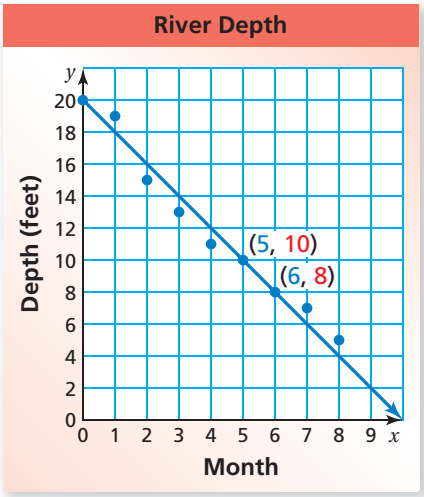
A **line of fit** is a line drawn on a scatter plot close to most of the data points. It can be used to estimate data on a graph.

EXAMPLE 1 Finding a Line of Fit

Month, x	Depth (feet), y
0	20
1	19
2	15
3	13
4	11
5	10
6	8
7	7
8	5

The table shows the depth of a river x months after a monsoon season ends. (a) Make a scatter plot of the data and draw a line of fit. (b) Write an equation of the line of fit. (c) Interpret the slope and the y-intercept of the line of fit. (d) Predict the depth in month 9.

a. Plot the points in a coordinate plane. The scatter plot shows a negative linear relationship. Draw a line that is close to the data points. Try to have as many points above the line as below it.



b. The line passes through (5, 10) and (6, 8).

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{-2}{1} = -2$$

Because the line crosses the y-axis at (0, 20), the y-intercept is 20.

∴ So, an equation of the line of fit is $y = -2x + 20$.

c. The slope is -2, and the y-intercept is 20. So, the depth of the river is 20 feet at the end of the monsoon season and decreases by about 2 feet per month.

Study Tip
A line of fit does not need to pass through any of the data points.

d. To predict the depth in month 9, substitute 9 for x in the equation of the line of fit.

$$y = -2x + 20 = -2(9) + 20 = 2$$

∴ The depth in month 9 should be about 2 feet.

On Your Own

Now You're Ready
Exercises 5 and 6

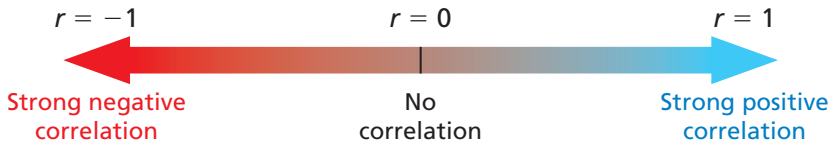
1. The table shows the numbers of people who have attended a festival over an 8-year period. (a) Make a scatter plot of the data and draw a line of fit. (b) Write an equation of the line of fit. (c) Interpret the slope and the y-intercept of the line of fit. (d) Predict the number of people who will attend the festival in year 10.

Year, x	1	2	3	4	5	6	7	8
Attendance, y	420	500	650	900	1100	1500	1750	2400

Study Tip

You know how to use two points to find an equation of a line of fit. When finding an equation of the line of best fit, every point in the data set is used.

Graphing calculators use a method called *linear regression* to find a precise line of fit called a **line of best fit**. This line best models a set of data. A calculator often gives a value r called the *correlation coefficient*. This value tells whether the correlation is positive or negative, and how closely the equation models the data. Values of r range from -1 to 1. When r is close to 1 or -1, there is a strong correlation between the variables. As r gets closer to 0, the correlation becomes weaker.

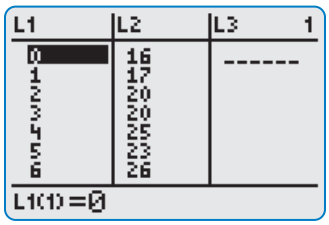


EXAMPLE 2 Finding a Line of Best Fit Using Technology

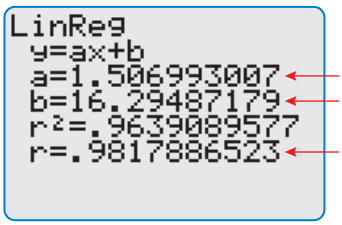
The table shows the worldwide movie ticket sales y (in billions of dollars) from 2000 to 2011, where $x = 0$ represents the year 2000. Use a graphing calculator to find an equation of the line of best fit. Identify and interpret the correlation coefficient.

Year, x	0	1	2	3	4	5	6	7	8	9	10	11
Ticket Sales, y	16	17	20	20	25	23	26	26	28	29	32	33

Step 1: Enter the data from the table into your calculator.



Step 2: Use the *linear regression* feature.

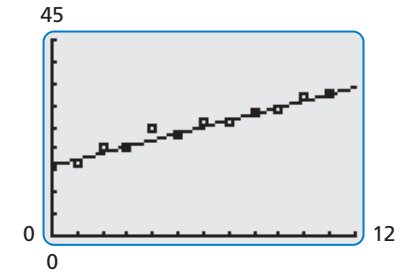


Study Tip

The slope of 1.5 indicates that sales are increasing by about \$1.5 billion each year. The y-intercept of 16 represents the ticket sales of \$16 billion for 2000.

An equation of the line of best fit is $y = 1.5x + 16$. The correlation coefficient is about 0.982. This means that the relationship between years and ticket sales is a strong positive correlation and that the equation closely models the data.

Check Use a graphing calculator to make a scatter plot and graph the line of best fit.



On Your Own

Now You're Ready
Exercises 8–10

2. Use a graphing calculator to find an equation of the line of best fit for the data in Example 1. Identify and interpret the correlation coefficient.

9.2 Exercises

Check It Out
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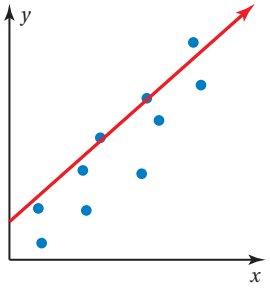
Vocabulary and Concept Check

1. **WRITING** Explain why a line of fit is helpful when analyzing data.
2. **REASONING** Tell whether the line drawn on the graph is a good fit for the data. Explain your reasoning.
3. **NUMBER SENSE** Which correlation coefficient indicates a stronger relationship: -0.98 or 0.91? Explain.



Practice and Problem Solving

4. **BLUEBERRIES** The table shows the weights y of x pints of blueberries.

Number of Pints, x	0	1	2	3	4	5
Weight (pounds), y	0	0.8	1.50	2.20	3.0	3.75

   a. Graph the data in the table.
   b. Draw a line that you think best approximates the points.
   c. Write an equation for your line.
   d. Use the equation to predict the weight of 10 pints of blueberries.
   e. Blueberries cost \$2.25 per pound. How much do 10 pints of blueberries cost?

1 5. **HOT CHOCOLATE** The table shows the daily high temperature (°F) and the number of hot chocolates sold at a coffee shop for eight randomly selected days.

Temperature (°F), x	30	36	44	51	60	68	75	82
Hot Chocolates, y	45	43	36	35	30	27	23	17

   a. Make a scatter plot of the data and draw a line of fit.
   b. Write an equation of the line of fit.
   c. Interpret the slope and the y-intercept of the line of fit.
   d. Predict the number of hot chocolates sold when the high temperature is 20°F.

6. **VACATION** The table shows the distance you are away from home over a 6-hour period of your vacation.

Hours, x	Distance (miles), y
1	62
2	123
3	188
4	228
5	280
6	344

   a. Make a scatter plot of the data and draw a line of fit.
   b. Write an equation of the line of fit.
   c. About how many miles per hour do you travel?
   d. About how far were you from home when you started?
   e. Predict the distance from home in 7 hours.

7. REASONING A data set has no relationship. Is it possible to find a line of fit for the data? Explain.

2 8. AMUSEMENT PARK The table shows the attendance y (in thousands) at an amusement park from 2004 to 2013, where $x = 4$ represents the year 2004. Use a graphing calculator to find an equation of the line of best fit. Identify and interpret the correlation coefficient.

Year, x	4	5	6	7	8	9	10	11	12	13
Attendance (thousands), y	850	845	828	798	800	792	785	781	775	760

9. SNOWSTORM The table shows the total snow depth y (in inches) on the ground during a snowstorm x hours after it began. Use a graphing calculator to find an equation of the line of best fit. Identify and interpret the correlation coefficient. Use your equation to estimate how much snow was on the ground before the snowstorm began.

Hours, x	1	2	3	4	5	6	7	8
Snow Depth (inches), y	5	6	6.75	7.75	8.5	9.5	10.5	11.5

10. TEXTING The table shows the numbers y (in billions) of text messages sent from 2006 to 2011, where $x = 6$ represents the year 2006.

Year, x	Text Messages (billions), y
6	113
7	241
8	601
9	1360
10	1806
11	2206

a. Use a graphing calculator to find an equation of the line of best fit. Identify and interpret the correlation coefficient.

b. Interpret the slope of the line of best fit. Does the y-intercept make sense for this problem? Explain.

c. Predict the number of text messages sent in 2015.

11. Modeling The table shows the height y (in feet) of a baseball x seconds after it was hit.

Seconds, x	Height (feet), y
0	3
0.5	39
1	67
1.5	87
2	99

a. Use a graphing calculator to find an equation of the line of best fit. Identify and interpret the correlation coefficient.

b. Predict the height after 5 seconds.

c. The actual height after 5 seconds is about 3 feet. Why do you think this is different from your prediction?

Fair Game Review What you learned in previous grades & lessons

Write the decimal as a fraction or a mixed number. *(Section 7.4)*

12. $0.\overline{2}$ **13.** $-2.\overline{7}$ **14.** $-1.4\overline{6}$ **15.** $0.\overline{81}$

16. MULTIPLE CHOICE Which expression represents the volume of a sphere with radius r? *(Section 8.3)*

(A) $\frac{1}{3}\pi r^2 h$ (B) $\pi r^2 h$ (C) $4\pi r^2$ (D) $\frac{4}{3}\pi r^3$

9.1–9.2 Quiz

Check It Out
Progress Check
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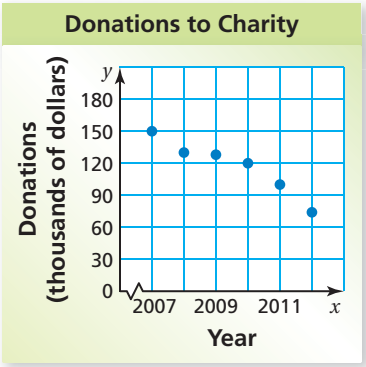
1. CHARITY The scatter plot shows the amount of money donated to a charity from 2007 to 2012. *(Section 9.1)*



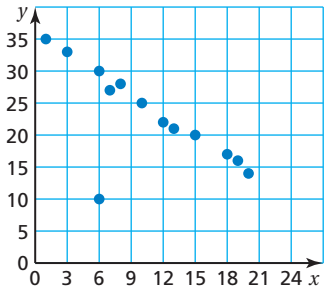
a. In what year did the charity receive $150,000?

b. How much did the charity receive in 2010?

c. Describe the relationship shown by the data.

Describe the relationship between the data. Identify any outliers, gaps, or clusters. *(Section 9.1)*

2.



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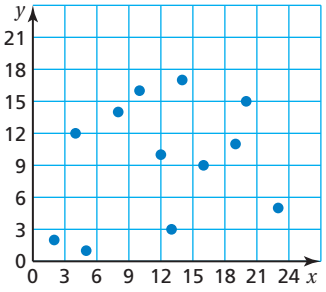

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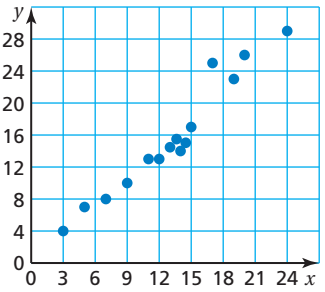


Year, x	Millions of Customers, y
0	12
1	18
2	21
3	28
4	33
5	38
6	42
7	48

5. CUSTOMERS SERVED The table shows the numbers of customers (in millions) served by a restaurant chain over an 8-year period. *(Section 9.1 and Section 9.2)*

a. Make a scatter plot of the data. Then describe the relationship between the data. Identify any outliers, gaps, or clusters.

b. Use a graphing calculator to find the equation of the line of best fit for the data. Identify and interpret the correlation coefficient.

6. CATS An animal shelter opens in December. The table shows the number of cats adopted from the shelter each month from January to September. *(Section 9.2)*

Month	1	2	3	4	5	6	7	8	9
Cats	3	6	7	11	13	14	15	18	19

a. Make a scatter plot of the data and draw a line of fit.

b. Write an equation of the line of fit.

c. Interpret the slope and the y-intercept of the line of fit.

d. Predict how many cats will be adopted in October.

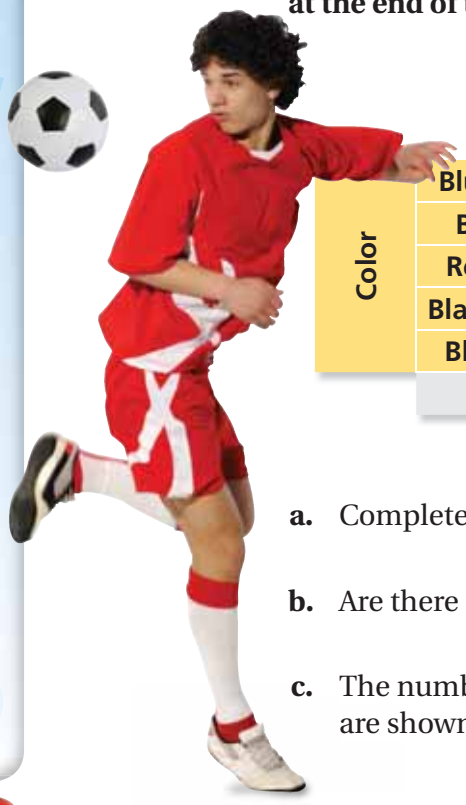
9.3 Two-Way Tables

Essential Question How can you read and make a two-way table?

Two categories of data can be displayed in a *two-way table*.

1 ACTIVITY: Reading a Two-Way Table

Work with a partner. You are the manager of a sports shop. The two-way table shows the numbers of soccer T-shirts that your shop has left in stock at the end of the season.



Color	T-Shirt Size S	M	L	XL	XXL	Total
Blue/White	5	4	1	0	2	
Blue/Gold	3	6	5	2	0	
Red/White	4	2	4	1	3	
Black/White	3	4	1	2	1	
Black/Gold	5	2	3	0	2	
Total						65

a. Complete the totals for the rows and columns.

b. Are there any black-and-gold XL T-shirts in stock? Justify your answer.

c. The numbers of T-shirts you ordered at the beginning of the season are shown below. Complete the two-way table.

Color	T-Shirt Size S	M	L	XL	XXL	Total
Blue/White	5	6	7	6	5	
Blue/Gold	5	6	7	6	5	
Red/White	5	6	7	6	5	
Black/White	5	6	7	6	5	
Black/Gold	5	6	7	6	5	
Total						

d. **REASONING** How would you alter the numbers of T-shirts you order for next season? Explain your reasoning.

Common Core

Data Analysis

In this lesson, you will

- read two-way tables.
- make and interpret two-way tables.

Learning Standard 8.SP.4

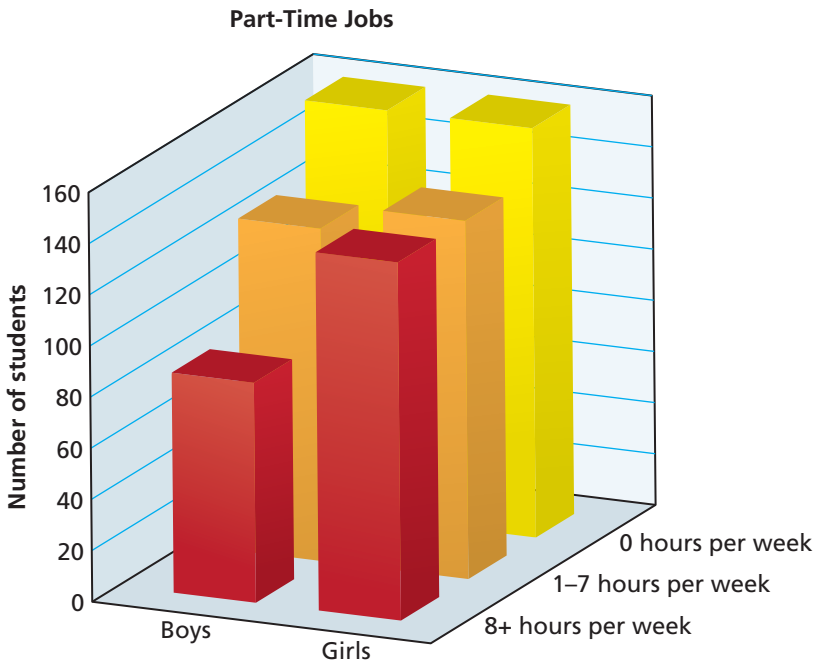
2 ACTIVITY: Analyzing Data

Math Practice 3

Construct Arguments

What are the advantages of using a table instead of a graph to analyze data?

Work with a partner. The three-dimensional two-way table shows information about the numbers of hours students at a high school work at part-time jobs during the school year.



a. Make a two-way table showing the data. Use estimation to find the entries in your table.

b. Write two observations you can make that summarize the data in your table.

c. **REASONING** A newspaper article claims that more boys than girls drop out of high school to work full-time. Do the data support this claim? Explain your reasoning.

What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you read and make a two-way table?

4. Find a real-life data set that you can represent by a two-way table. Then make a two-way table for the data set.

Practice Use what you learned about two-way tables to complete Exercises 3–6 on page 390.

9.3 Lesson

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Key Vocabulary
two-way table, *p. 388*
joint frequency, *p. 388*
marginal frequency, *p. 388*

A **two-way table** displays two categories of data collected from the same source.

You randomly survey students in your school about their grades on the last test and whether they studied for the test. The two-way table shows your results. Each entry in the table is called a **joint frequency**.

		Student	
		Studied	Did Not Study
Grade	Passed	21	2
	Failed	1	6

joint frequency

EXAMPLE 1 Reading a Two-Way Table

How many of the students in the survey above studied for the test and passed?

The entry in the "Studied" column and "Passed" row is 21.

∴ So, 21 of the students in the survey studied for the test and passed.

The sums of the rows and columns in a two-way table are called **marginal frequencies**.

EXAMPLE 2 Finding Marginal Frequencies

Find and interpret the marginal frequencies for the survey above.

Create a new column and a new row for the sums. Then add the entries.

		Student		
		Studied	Did Not Study	Total
Grade	Passed	21	2	23
	Failed	1	6	7
	Total	22	8	30

23 students passed.
7 students failed.
30 students were surveyed.
22 students studied.
8 students did not study.

On Your Own

Now You're Ready
Exercises 5–8

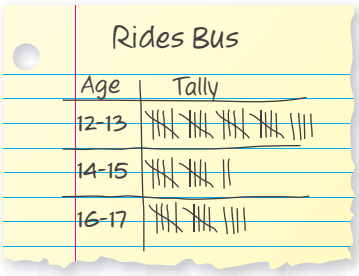
1. You randomly survey students in a cafeteria about their plans for a football game and a school dance. The two-way table shows your results.

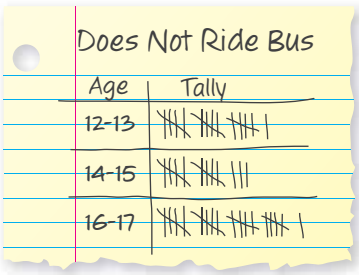
		Football Game	
		Attend	Not Attend
Dance	Attend	35	5
	Not Attend	16	20

   a. How many students will attend the dance but not the football game?

   b. Find and interpret the marginal frequencies for the survey.

EXAMPLE 3 Making a Two-Way Table





You randomly survey students between the ages of 12 and 17 about whether they ride the bus to school. The results are shown in the tally sheets. Make a two-way table that includes the marginal frequencies.

The two categories for the table are the ages and whether or not they ride the bus. Use the tally sheets to calculate each joint frequency. Then add to find each marginal frequency.

		Age			
		12–13	14–15	16–17	Total
Student	Rides Bus	24	12	14	50
	Does Not Ride Bus	16	13	21	50
	Total	40	25	35	100

EXAMPLE 4 Finding a Relationship in a Two-Way Table

Use the two-way table in Example 3.

a. For each age group, what percent of the students in the survey ride the bus to school? do not ride the bus to school? Organize the results in a two-way table. Explain what one of the entries represents.

		Age		
		12–13	14–15	16–17
Student	Rides Bus	60%	48%	40%
	Does Not Ride Bus	40%	52%	60%

$\frac{14}{35} = 0.4$
So, 40% of the 16- and 17-year-old students in the survey ride the bus to school.

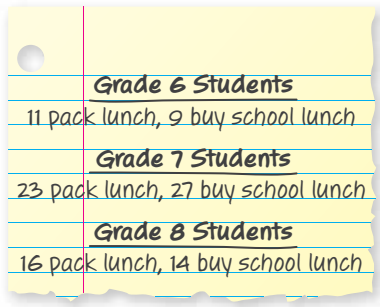
b. Does the table in part (a) show a relationship between age and whether students ride the bus to school? Explain.

∴ Yes, the table shows that as age increases, students are less likely to ride the bus to school.

On Your Own

Now You're Ready
Exercise 10

2. You randomly survey students in a school about whether they buy a school lunch or pack a lunch. Your results are shown.



a. Make a two-way table that includes the marginal frequencies.

b. For each grade level, what percent of the students in the survey pack a lunch? buy a school lunch? Organize the results in a two-way table. Explain what one of the entries represents.

c. Does the table in part (b) show a relationship between grade level and lunch choice? Explain.

9.3 Exercises

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Vocabulary and Concept Check

1. **VOCABULARY** Explain the relationship between joint frequencies and marginal frequencies.
2. **OPEN-ENDED** Describe how you can use a two-way table to organize data you collect from a survey.

Practice and Problem Solving

You randomly survey students about participating in their class's yearly fundraiser. You display the two categories of data in the two-way table.

3. Find the total of each row.
4. Find the total of each column.
5. How many female students will be participating in the fundraiser?
6. How many male students will *not* be participating in the fundraiser?

		Fundraiser	
		No	Yes
Gender	Female	22	51
	Male	30	29

Find and interpret the marginal frequencies.

7.

		School Play	
		Attend	Not Attend
Class	Junior	41	30
	Senior	52	23

8.

		Cell Phone Minutes	
		Limited	Unlimited
Text Plan	Limited	78	0
	Unlimited	175	15

9. **GOALS** You randomly survey students in your school. You ask what is most important to them: grades, popularity, or sports. You display your results in the two-way table.

   a. How many 7th graders chose sports? How many 8th graders chose grades?

   b. Find and interpret the marginal frequencies for the survey.

   c. What percent of students in the survey are 6th graders who chose popularity?

		Goal		
		Grades	Popularity	Sports
Grade	6th	31	18	23
	7th	39	16	19
	8th	42	6	17

3 **4** **10. SAVINGS** You randomly survey people in your neighborhood about whether they have at least $1000 in savings. The results are shown in the tally sheets.

Have at Least $1000 in Savings

Age	Tally
20-29	
30-39	
40-49	

Don't Have at Least $1000 in Savings

Age	Tally
20-29	
30-39	
40-49	

a. Make a two-way table that includes the marginal frequencies.

b. For each age group, what percent of the people have at least $1000 in savings? do not have at least $1000 in savings? Organize the results in a two-way table.

c. Does the table in part (b) show a relationship between age and whether people have at least $1000 in savings? Explain.

11. EYE COLOR You randomly survey students in your school about the color of their eyes. The results are shown in the tables.

Eye Color of Males Surveyed		
Green	Blue	Brown
5	16	27

Eye Color of Females Surveyed		
Green	Blue	Brown
3	19	18

a. Make a two-way table.

b. Find and interpret the marginal frequencies for the survey.

c. For each eye color, what percent of the students in the survey are male? female? Organize the results in a two-way table. Explain what two of the entries represent.

12. REASONING Use the information from Exercise 11. For each gender, what percent of the students in the survey have green eyes? blue eyes? brown eyes? Organize the results in a two-way table. Explain what two of the entries represent.

13. **Precision** What percent of students in the survey in Exercise 11 are either female or have green eyes? What percent of students in the survey are males who do not have green eyes? Find and explain the sum of these two percents.

Fair Game Review What you learned in previous grades & lessons

Write an equation of the line that passes through the points. *(Section 4.6)*

14. $(0, 1), (-2, -5)$ **15.** $(0, -2), (3, 13)$ **16.** $(-4, 1), (0, 3)$

17. MULTIPLE CHOICE Which equation does not represent a linear function? *(Section 6.4)*

Ⓐ $y = 4x$ Ⓑ $xy = 8$ Ⓒ $y = -3$ Ⓓ $6x + 5y = -2$

9.4 Choosing a Data Display

Essential Question How can you display data in a way that helps you make decisions?

1 ACTIVITY: Displaying Data

Work with a partner. Analyze and display each data set in a way that best describes the data. Explain your choice of display.

a. ROADKILL A group of schools in New England participated in a 2-month study. They reported 3962 dead animals.

Birds: 307
Mammals: 2746
Amphibians: 145
Reptiles: 75
Unknown: 689

b. BLACK BEAR ROADKILL The data below show the numbers of black bears killed on a state's roads from 1993 to 2012.

1993:	30	2000:	47	2007:	99
1994:	37	2001:	49	2008:	129
1995:	46	2002:	61	2009:	111
1996:	33	2003:	74	2010:	127
1997:	43	2004:	88	2011:	141
1998:	35	2005:	82	2012:	135
1999:	43	2006:	109		

c. RACCOON ROADKILL A 1-week study along a 4-mile section of road found the following weights (in pounds) of raccoons that had been killed by vehicles.

13.4	14.8	17.0	12.9
21.3	21.5	16.8	14.8
15.2	18.7	18.6	17.2
18.5	9.4	19.4	15.7
14.5	9.5	25.4	21.5
17.3	19.1	11.0	12.4
20.4	13.6	17.5	18.5
21.5	14.0	13.9	19.0

d. What do you think can be done to minimize the number of animals killed by vehicles?

Common Core

Data Analysis

In this lesson, you will

- choose appropriate data displays.
- identify and analyze misleading data displays.

Applying Standard 8.SP.1

2 ACTIVITY: Statistics Project

Math Practice 4

Use a Graph

How can you use a graph to represent the data you have gathered for your report? What does the graph tell you about the data?

ENDANGERED SPECIES PROJECT **Use the Internet or some other reference to write a report about an animal species that is (or has been) endangered. Include graphical displays of the data you have gathered.**

Sample: Florida Key Deer

In 1939, Florida banned the hunting of Key deer. The numbers of Key deer fell to about 100 in the 1940s.

In 1947, public sentiment was stirred by 11-year-old Glenn Allen from Miami. Allen organized Boy Scouts and others in a letter-writing campaign that led to the establishment of the National Key Deer Refuge in 1957. The approximately 8600-acre refuge includes 2280 acres of designated wilderness.

The Key Deer Refuge has increased the population of Key deer. A recent study estimated the total Key deer population to be approximately 800.

About half of Key deer deaths are due to vehicles.

One of two Key deer wildlife underpasses on Big Pine Key

What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you display data in a way that helps you make decisions? Use the Internet or some other reference to find examples of the following types of data displays.

- Bar graph
- Circle graph
- Scatter plot
- Stem-and-leaf plot
- Box-and-whisker plot

Practice Use what you learned about choosing data displays to complete Exercise 3 on page 397.

9.4 Lesson

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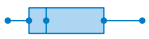
Key Idea

Data Display	What does it do?
Pictograph	shows data using pictures 
Bar Graph	shows data in specific categories 
Circle Graph	shows data as parts of a whole 
Line Graph	shows how data change over time 
Histogram	shows frequencies of data values in intervals of the same size 
Stem-and-Leaf Plot	orders numerical data and shows how they are distributed 
Box-and-Whisker Plot	shows the variability of a data set by using quartiles 
Dot Plot	shows the number of times each value occurs in a data set 
Scatter Plot	shows the relationship between two data sets by using ordered pairs in a coordinate plane 

EXAMPLE 1 Choosing an Appropriate Data Display

Choose an appropriate data display for the situation. Explain your reasoning.

a. the number of students in a marching band each year

A line graph shows change over time. So, a line graph is an appropriate data display.

b. a comparison of people's shoe sizes and their heights

You want to compare two different data sets. So, a scatter plot is an appropriate data display.

On Your Own

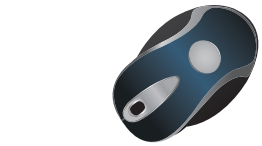
Now You're Ready
Exercises 4–7

Choose an appropriate data display for the situation. Explain your reasoning.

1. the population of the United States divided into age groups
2. the percents of students in your school who play basketball, football, soccer, or lacrosse

EXAMPLE 2 Identifying an Appropriate Data Display

You record the number of hits for your school's new website for 5 months. Tell whether the data display is appropriate for representing how the number of hits changed during the 5 months. Explain your reasoning.

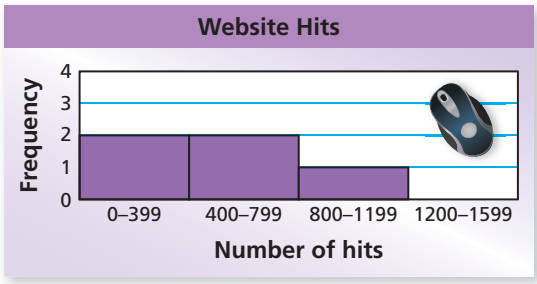


Month	Hits
August	250
September	320
October	485
November	650
December	925

a.

The bar graph shows the number of hits for each month. So, it is an appropriate data display.

b.



The histogram does not show the number of hits for each month or how the number of hits changes over time. So, it is *not* an appropriate data display.

c.

The line graph shows how the number of hits changes over time. So, it is an appropriate data display.

On Your Own

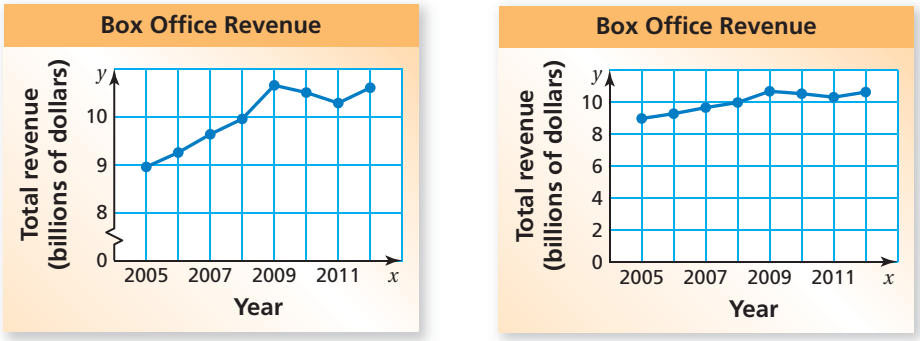
Now You're Ready
Exercises 8 and 9

Tell whether the data display is appropriate for representing the data in Example 2. Explain your reasoning.

3. dot plot **4.** circle graph **5.** stem-and-leaf plot

EXAMPLE 3 Identifying a Misleading Data Display

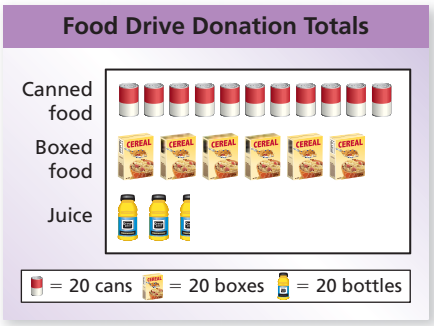
Which line graph is misleading? Explain.



The vertical axis of the line graph on the left has a break (≑) and begins at 8. This graph makes it appear that the total revenue increased rapidly from 2005 to 2009. The graph on the right has an unbroken axis. It is more honest and shows that the total revenue increased slowly.

∴ So, the graph on the left is misleading.

EXAMPLE 4 Analyzing a Misleading Data Display



A volunteer concludes that the numbers of cans of food and boxes of food donated were about the same. Is this conclusion accurate? Explain.

Each icon represents the same number of items. Because the box icon is larger than the can icon, it looks like the number of boxes is about the same as the number of cans. But the number of boxes is actually about half of the number of cans.

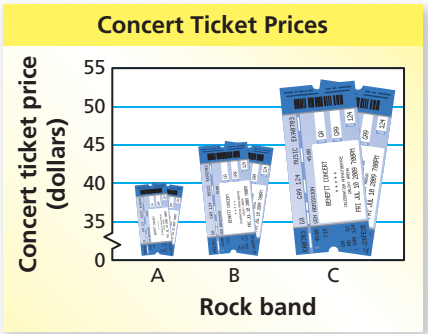
∴ So, the conclusion is not accurate.

On Your Own

Now You're Ready
Exercises 11–14

Explain why the data display is misleading.

6.



7.

9.4 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **REASONING** Can more than one display be appropriate for a data set? Explain.
2. **OPEN-ENDED** Describe how a histogram can be misleading.

Practice and Problem Solving

3. Analyze and display the data in a way that best describes the data. Explain your choice of display.

Notebooks Sold in One Week				
192 red	170 green	203 black	183 pink	230 blue
165 yellow	210 purple	250 orange	179 white	218 other

Choose an appropriate data display for the situation. Explain your reasoning.

1 4. a student's test scores and how the scores are spread out

5. the distance a person drives each month

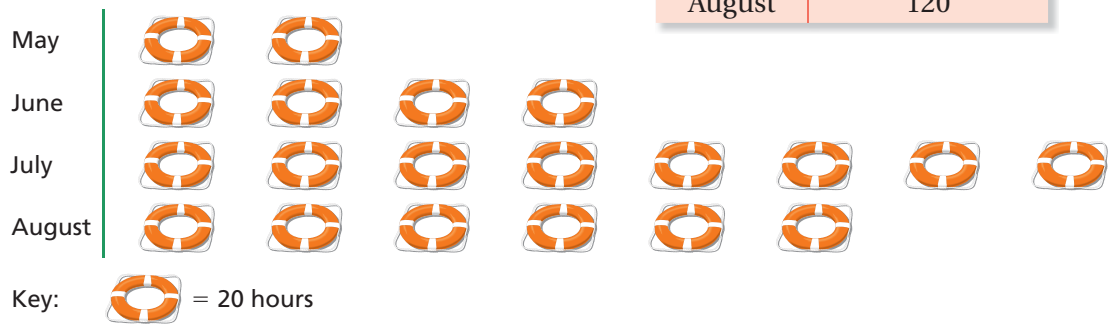
6. the outcome of rolling a number cube

7. homework problems assigned each day

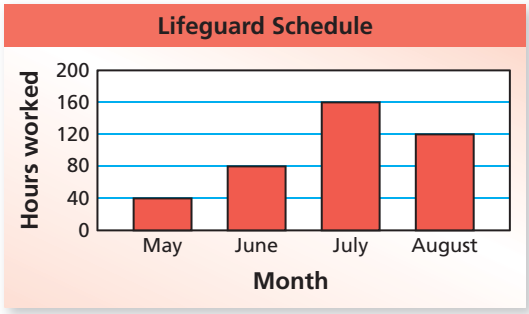
2 8. **LIFEGUARD** The table shows how many hours you worked as a lifeguard from May to August. Tell whether the data display is appropriate for representing how the number of hours worked changed during the 4 months. Explain your reasoning.

Lifeguard Schedule	
Month	Hours Worked
May	40
June	80
July	160
August	120

a.



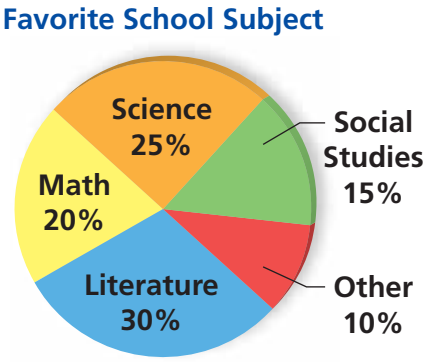
b.



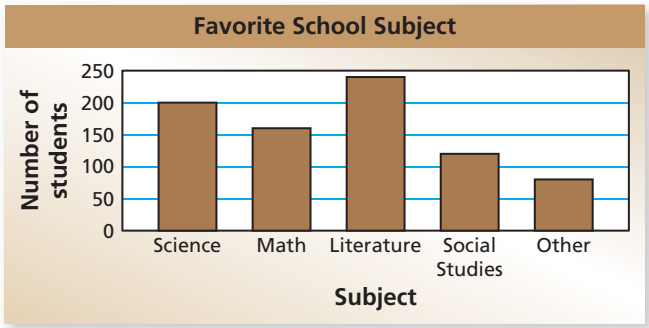
9. FAVORITE SUBJECT A survey asked 800 students to choose their favorite subject. The results are shown in the table. Tell whether the data display is appropriate for representing the portion of students who prefer math. Explain your reasoning.

Favorite School Subject	
Subject	**Number of Students**
Science	200
Math	160
Literature	240
Social Studies	120
Other	80

a.



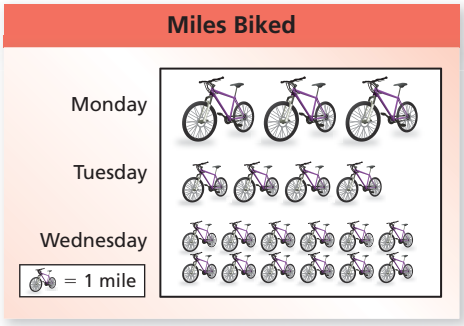
b.



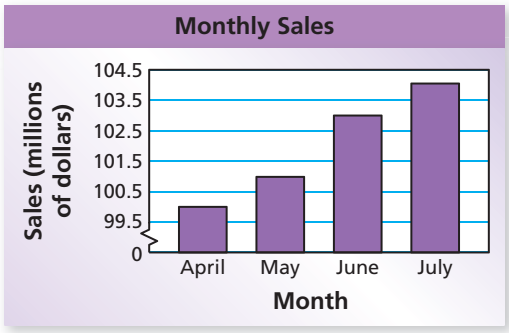
10. WRITING When should you use a histogram instead of a bar graph to display data? Use an example to support your answer.

Explain why the data display is misleading.

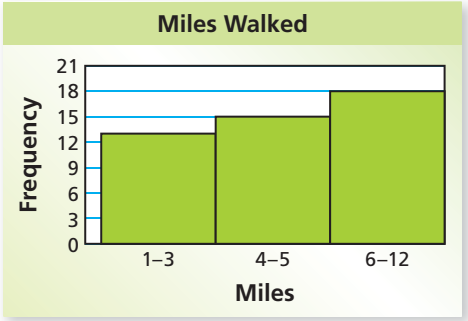
3 4 **11.**



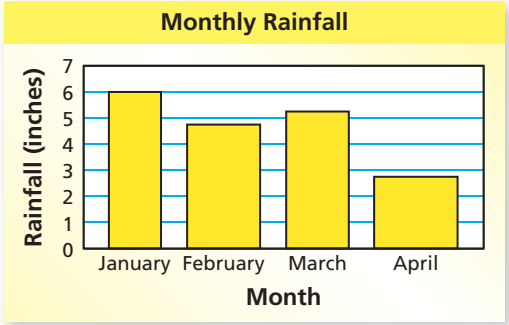
12.



13.

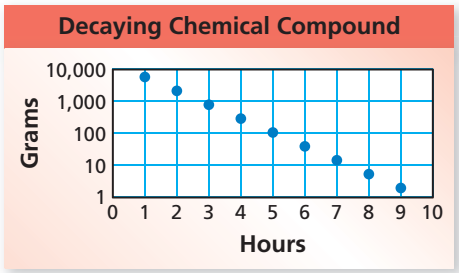


14.



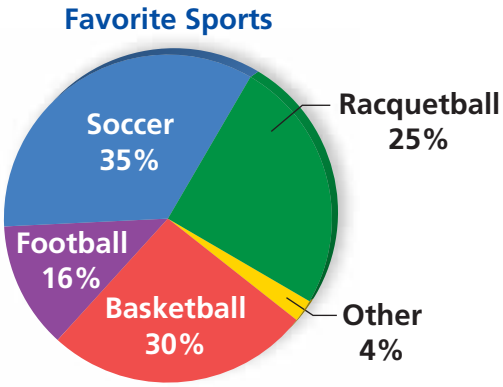
15. VEGETABLES A nutritionist wants to use a data display to show the favorite vegetables of the students at a school. Choose an appropriate data display for the situation. Explain your reasoning.

16. CHEMICALS A scientist gathers data about a decaying chemical compound. The results are shown in the scatter plot. Is the data display misleading? Explain.



17. REASONING What type of data display is appropriate for showing the mode of a data set?

18. SPORTS A survey asked 100 students to choose their favorite sports. The results are shown in the circle graph.



a. Explain why the graph is misleading.

b. What type of data display would be more appropriate for the data? Explain.

19. Structure With the help of computers, mathematicians have computed and analyzed billions of digits of the irrational number π. One of the things they analyze is the frequency of each of the numbers 0 through 9. The table shows the frequency of each number in the first 100,000 digits of π.

a. Display the data in a bar graph.

b. Display the data in a circle graph.

c. Which data display is more appropriate? Explain.

d. Describe the distribution.

Number	0	1	2	3	4	5	6	7	8	9
Frequency	9999	10,137	9908	10,025	9971	10,026	10,029	10,025	9978	9902

Fair Game Review What you learned in previous grades & lessons

Estimate the square root to the nearest (a) integer and (b) tenth. *(Section 7.4)*

20. $\sqrt{20}$ **21.** $-\sqrt{74}$ **22.** $\sqrt{140}$

23. MULTIPLE CHOICE What is 20% of 25% of 400? *(Skills Review Handbook)*

(A) 20 (B) 200 (C) 240 (D) 380

9.3–9.4 Quiz

Check It Out
Progress Check
BigIdeasMath.com

1. RECYCLING The results of a recycling survey are shown in the two-way table. Find and interpret the marginal frequencies. *(Section 9.3)*

		Recycle	
		Yes	No
Gender	Female	28	9
	Male	24	14

2. MUSIC The results of a music survey are shown in the two-way table. Find and interpret the marginal frequencies. *(Section 9.3)*

		Jazz	
		Likes	Dislikes
Country	Likes	26	14
	Dislikes	17	8

3. ELECTION The results of a voting survey are shown in the two-way table. *(Section 9.3)*

		Voter's Age		
		18–34	35–64	65+
Candidate	Smith	36	25	6
	Jackson	12	32	24

a. Find and interpret the marginal frequencies.

b. For each age group, what percent of voters prefer Smith? prefer Jackson? Organize your results in a two-way table.

c. Does your table in part (b) show a relationship between age and candidate preference? Explain.

Choose an appropriate data display for the situation. Explain your reasoning. *(Section 9.4)*

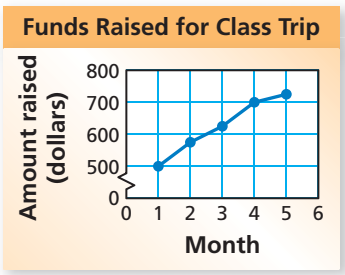
4. the percent of band students in each section of instruments

5. a company's profit for each week

6. TURTLES The tables show the weights (in pounds) of turtles caught in two ponds. Which type of data display would you use for this information? Explain. *(Section 9.4)*

Pond A			
12	13	15	6
7	8	12	7

Pond B			
9	12	5	8
12	15	16	19



7. FUNDRAISER The line graph shows the amount of money that the eighth-grade students at a school raised each month to pay for a class trip. Is the graph misleading? Explain. *(Section 9.4)*

9 Chapter Review

Check It Out
Vocabulary Help
BigIdeasMath.com

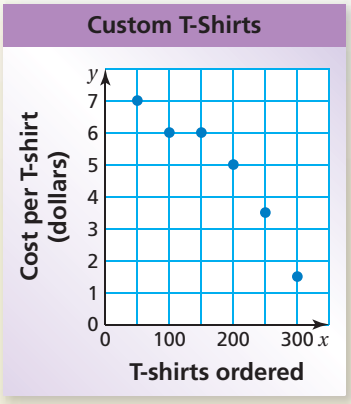
Review Key Vocabulary

scatter plot, *p. 374*
line of fit, *p. 380*
line of best fit, *p. 381*
two-way table, *p. 388*
joint frequency, *p. 388*
marginal frequency, *p. 388*

Review Examples and Exercises

9.1 Scatter Plots *(pp. 372–377)*

Your school is ordering custom T-shirts. The scatter plot shows the number of T-shirts ordered and the cost per shirt. What tends to happen to the cost per shirt as the number of T-shirts ordered increases?

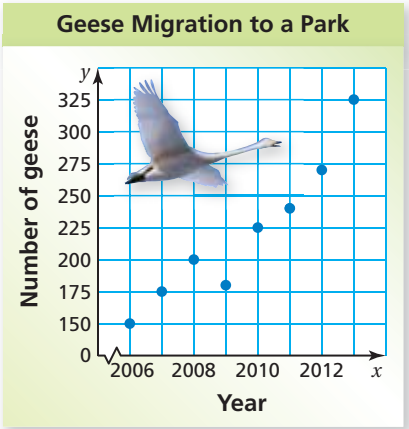


Looking at the graph, the plotted points go down from left to right.

So, as the number of T-shirts ordered increases, the cost per shirt decreases.

Exercises

1. **MIGRATION** The scatter plot shows the number of geese that migrated to a park each season.

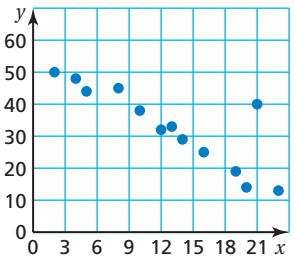


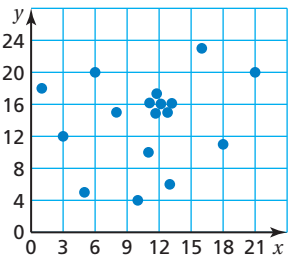
   a. In what year did 270 geese migrate?

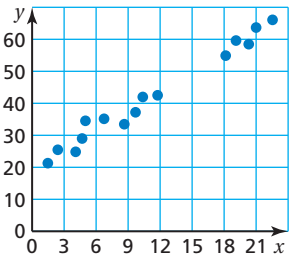
   b. How many geese migrated in 2010?

   c. Describe the relationship shown by the data.

Describe the relationship between the data. Identify any outliers, gaps, or clusters.

2. 

3. 

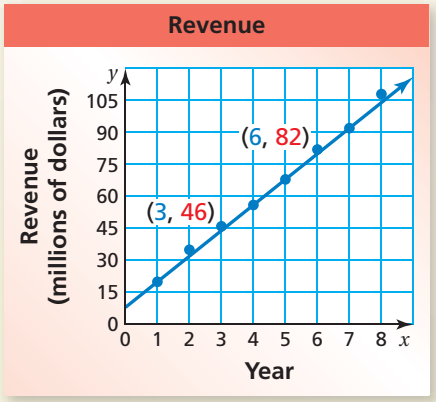
4. 

9.2 Lines of Fit *(pp. 378–383)*

The table shows the revenue (in millions of dollars) for a company over an 8-year period. (a) Make a scatter plot of the data and draw a line of fit. (b) Write an equation of the line of fit. (c) Interpret the slope and the *y*-intercept of the line of fit. (d) Predict what the revenue will be in year 9.

Year, x	1	2	3	4	5	6	7	8
Revenue (millions of dollars), y	20	35	46	56	68	82	92	108

a. Plot the points in a coordinate plane. The scatter plot shows a positive linear relationship. Draw a line that is close to the data points.



b. slope $= \frac{\text{rise}}{\text{run}} = \frac{36}{3} = 12$

Because the line crosses the *y*-axis at (0, 8), the *y*-intercept is 8.

∴ So, an equation of the line of fit is $y = 12x + 8$.

c. The slope is 12. So, the revenue increased by about \$12 million each year. The *y*-intercept is 8. So, you can estimate that the revenue was \$8 million in the year before this 8-year period.

d. $y = 12x + 8 = 12(9) + 8 = 116$

∴ The revenue in year 9 will be about \$116 million.

Exercises

5. STUDENTS The table shows the number of students at a middle school over a 10-year period.

Year, x	Number of Students, y
1	492
2	507
3	520
4	535
5	550
6	562
7	577
8	591
9	604
10	618

a. Make a scatter plot of the data and draw a line of fit.

b. Write an equation of the line of fit.

c. Interpret the slope and the *y*-intercept of the line of fit.

d. Predict the number of students in year 11.

6. LINE OF BEST FIT Use a graphing calculator to find an equation of the line of best fit for the data in Exercise 5. Identify and interpret the correlation coefficient.

9.3 Two-Way Tables *(pp. 386–391)*

You randomly survey students in your school about whether they liked a recent school play. The results are shown. Make a two-way table that includes the marginal frequencies. What percent of the students surveyed liked the play?

Male Students
48 likes, 12 dislikes

Female Students
56 likes, 14 dislikes

Of the 130 students surveyed, 104 students liked the play.

∴ Because $\frac{104}{130} = 0.8$, 80% of the students in the survey liked the play.

		Student		
		Liked	Did Not Like	Total
Gender	Male	48	12	60
	Female	56	14	70
	Total	104	26	130

Exercises

You randomly survey people at a mall about whether they like the new food court. The results are shown.

Teenagers
96 likes, 4 dislikes

Adults
21 likes, 79 dislikes

Senior Citizens
18 likes, 82 dislikes

7. Make a two-way table that includes the marginal frequencies.

8. For each group, what percent of the people surveyed like the food court? dislike the food court? Organize your results in a two-way table.

9. Does your table in Exercise 8 show a relationship between age and whether people like the food court?

9.4 Choosing a Data Display *(pp. 392–399)*

Choose an appropriate data display for the situation. Explain your reasoning.

a. the percent of votes that each candidate received in an election

∴ A circle graph shows data as parts of a whole. So, a circle graph is an appropriate data display.

b. the distribution of the ages of U.S. presidents

∴ A stem-and-leaf plot orders numerical data and shows how they are distributed. So, a stem-and-leaf plot is an appropriate data display.

Exercises

Choose an appropriate data display for the situation. Explain your reasoning.

10. the number of pairs of shoes sold by a store each week

11. a comparison of the heights of brothers and sisters

9 Study Help

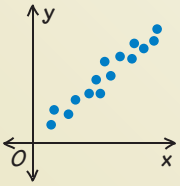
Check It Out
Graphic Organizer
BigIdeasMath.com

You can use an **information frame** to help you organize and remember concepts. Here is an example of an information frame for scatter plots.

Scatter Plots

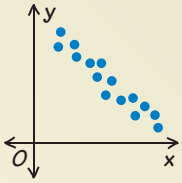
Words:
A scatter plot shows the relationship between two data sets using ordered pairs in a coordinate plane.

Example:
Positive Linear Relationship



The points lie close to a line. As x increases, y increases.

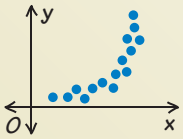
Example:
Negative Linear Relationship



The points lie close to a line. As x increases, y decreases.

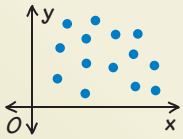
Examples:

Nonlinear Relationship



The points lie in the shape of a curve.

No Relationship



The points show no pattern.

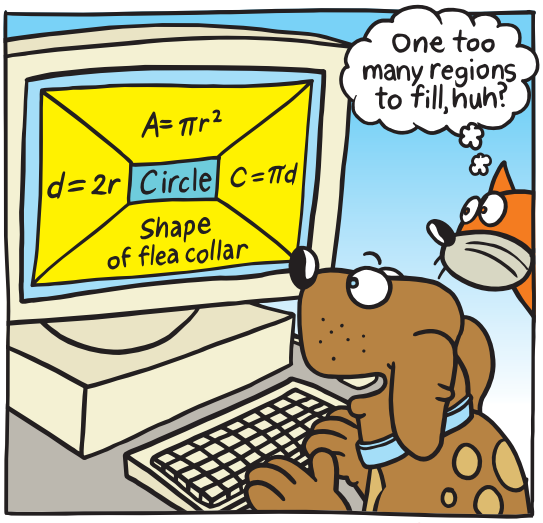
On Your Own

Make an information frame to help you study this topic.

1. lines of fit

After you complete this chapter, make information frames for the following topics.

2. two-way tables
3. data displays

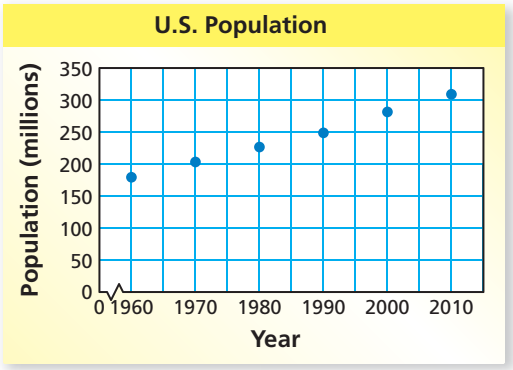


"Dear Teacher, I am emailing my information frame showing the characteristics of circles."

9 Chapter Test

Check It Out
Test Practice
BigIdeasMath.com

1. POPULATION The graph shows the population (in millions) of the United States from 1960 to 2010.



a. In what year was the population of the United States about 180 million?

b. What was the approximate population of the United States in 1990?

c. Describe the trend shown by the data.

2. WEIGHT The table shows the weight of a baby over several months.

Age (months)	Weight (pounds)
1	8
2	9.25
3	11.75
4	13
5	14.5
6	16

a. Make a scatter plot of the data and draw a line of fit.

b. Write an equation of the line of fit.

c. Interpret the slope and the y-intercept of the line of fit.

d. Predict how much the baby will weigh at 7 months.

		Nonfiction	
		Likes	Dislikes
Fiction	Likes	26	20
	Dislikes	22	2

3. READING You randomly survey students at your school about what type of books they like to read. The two-way table shows your results. Find and interpret the marginal frequencies.

Choose an appropriate data display for the situation. Explain your reasoning.

4. magazine sales grouped by price

5. the distance a person hikes each week

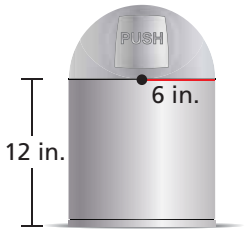
6. SAT The table shows the numbers y of students (in thousands) who took the SAT from 2006 to 2010, where $x = 6$ represents the year 2006. Use a graphing calculator to find an equation of the line of best fit. Identify and interpret the correlation coefficient.

Year, x	6	7	8	9	10
Number of Students, y	1466	1495	1519	1530	1548

7. RECYCLING You randomly survey shoppers at a supermarket about whether they use reusable bags. Of 60 male shoppers, 15 use reusable bags. Of 110 female shoppers, 60 use reusable bags. Organize your results in a two-way table. Include the marginal frequencies.

9 Standards Assessment

1. What is the volume of the trash bin? *(8.G.9)*



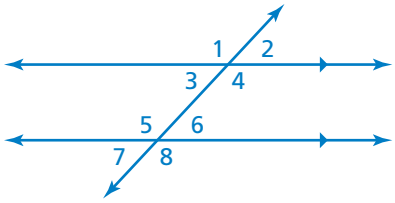
A. 288π in.^{3}

B. 576π in.^{3}

C. 648π in.^{3}

D. 720π in.^{3}

2. The diagram below shows parallel lines cut by a transversal. Which angle is the corresponding angle for $\angle 6$? *(8.G.5)*



F. $\angle 2$

G. $\angle 3$

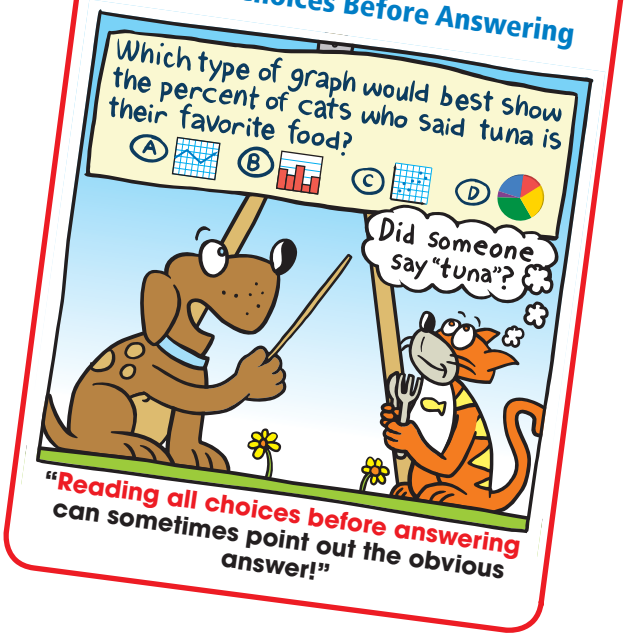
H. $\angle 4$

I. $\angle 8$

3. You randomly survey students in your school. You ask whether they have jobs. You display your results in the two-way table. How many male students do *not* have a job? *(8.SP.4)*

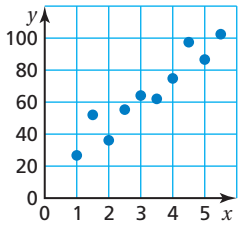
		Job	
		Yes	No
Gender	Male	27	12
	Female	31	17

Test-Taking Strategy
Read All Choices Before Answering

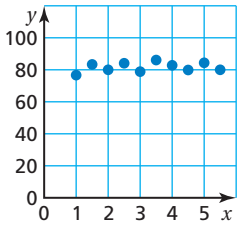


4. Which scatter plot shows a negative relationship between x and y? *(8.SP.1)*

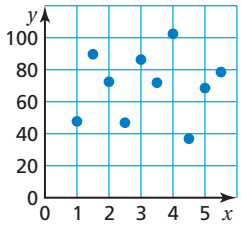
A.



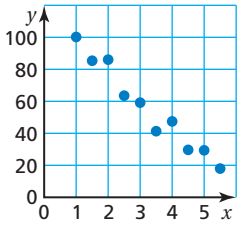
C.



B.



D.



5. The legs of a right triangle have the lengths of 8 centimeters and 15 centimeters. What is the length of the hypotenuse, in centimeters? *(8.G.7)*

6. What is the solution of the equation? *(8.EE.7b)*

$$0.22(x + 6) = 0.2x + 1.8$$

F. $x = 2.4$

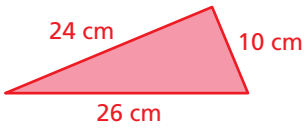
G. $x = 15.6$

H. $x = 24$

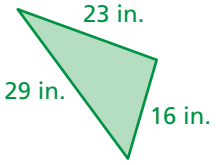
I. $x = 156$

7. Which triangle is *not* a right triangle? *(8.G.6)*

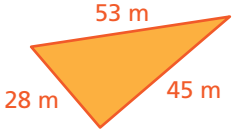
A.



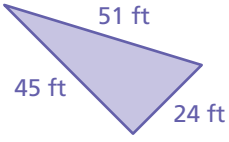
C.



B.



D.



8. A store has recorded total dollar sales each month for the past three years. Which type of graph would best show how sales have increased over this time period? *(8.SP.1)*

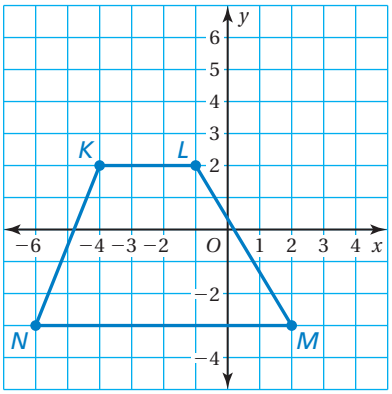
F. circle graph

G. line graph

H. histogram

I. stem-and-leaf plot

9. Trapezoid *KLMN* is graphed in the coordinate plane shown.



Rotate Trapezoid *KLMN* 90° clockwise about the origin. What are the coordinates of point *M′*, the image of point *M* after the rotation? *(8.G.3)*

A. (−3, −2)

B. (−2, −3)

C. (−2, 3)

D. (3, 2)

10. The table shows the numbers of hours students spent watching television from Monday through Friday for one week and their scores on a test that Friday. *(8.SP.1, 8.SP.2)*

Think
Solve
Explain

Hours of Television, *x*	5	2	10	15	3	4	8	2	12	9
Test Score, *y*	92	98	79	66	97	88	82	95	72	81

Part A Make a scatter plot of the data.

Part B Describe the relationship between hours of television watched and test score.

Part C Explain how to justify your answer in Part B using the linear regression feature of a graphing calculator.